

# Dynamics and Kinetics. Exercises 5: Solutions

## Problem 1



a) Steady state for  $ES$ :

$$0 \approx \frac{d[ES]}{dt} = k_1[E][S] + k_{-2}[E][P] - (k_{-1} + k_2)[ES]$$

$$[ES] = \frac{k_1[S] + k_{-2}[P]}{k_{-1} + k_2} [E] = \left( \frac{[S]}{K_S} + \frac{[P]}{K_P} \right) [E] = x[E],$$

where we define:

$$x := \frac{[S]}{K_S} + \frac{[P]}{K_P}$$

Rate of reaction:

$$v = \frac{d[P]}{dt} = k_2[ES] - k_{-2}[E][P] = \left( \frac{k_2}{K_S} [S] + \frac{k_2 k_{-2} - k_{-2}(k_{-1} + k_2)}{k_{-1} + k_2} [P] \right) [E]$$

$$= \left( \frac{k_2[S]}{K_S} - \frac{k_{-1}[P]}{K_P} \right) [E]$$

Total enzyme:  $[E]_0 = [E] + [ES] = [E](1 + x)$

$$v = \left( \frac{k_2[S]}{K_S} - \frac{k_{-1}[P]}{K_P} \right) \frac{[E]_0}{1 + x}$$

b) At short times:  $[P] \rightarrow 0$  (no product formed yet)

$$v = \left( \frac{\frac{k_2[S]}{K_S}}{1 + \frac{[S]}{K_S}} \right) [E]_0 = \frac{k_2[E]_0}{1 + \frac{K_S}{[S]}}$$

which corresponds to the Michaelis-Menten equation with  $K_M = K_S$  and  $v_{max} = k_2[E]_0$

c) Competitive inhibition



Steady state approximation:

$$0 \approx \frac{d[EI]}{dt} = k_3[E][I] - k_{-3}[EI] \quad \Rightarrow \quad [EI] = \frac{k_3}{k_{-3}} [E][I]$$

Total enzyme:

$$[E]_0 = [E] + [ES] + [EI] = [E](1 + x + y)$$

where we defined:

$$y := \frac{k_3}{k_{-3}}[I]$$

As in a)

$$\begin{aligned} \text{Rate } v &= \frac{d[P]}{dt} = \left( \frac{k_2[S]}{K_S} - \frac{k_{-1}[P]}{K_P} \right) [E] \\ &= \frac{\frac{k_2[S]}{K_S} - \frac{k_{-1}[P]}{K_P}}{1 + x + y} [E]_0 \end{aligned}$$

For short times ( $[P] \rightarrow 0$ ):

$$v = \frac{k_2[S][E]_0}{K_S + [S] + \frac{k_3}{k_{-3}} K_S [I]}$$

- d) Short times  $\Rightarrow$  we can consider only the simplified mechanisms for short times [either the Michaelis-Menten in part (b) or the Michaelis-Menten with inhibition in second part of part (c)]

Use: Lineweaver-Burk plots  $\left( \frac{1}{v} \text{ vs. } \frac{1}{[S]} \right)$

Recall from lecture:

i) Competitive inhibition  $\rightarrow$  changes slope,

ii) Incompetitive inhibition  $\rightarrow$  changes  $y$  intercept.

Results from linear regression:

Without inhibitor:  $\frac{1}{v} = a_0 \frac{1}{[S]} + b_0$        $a_0 = 4.0\text{s}$ ,    $b_0 = 5000\text{M}^{-1}\text{s}$

With inhibitor:  $\frac{1}{v} = a \frac{1}{[S]} + b$        $a = 11.5\text{s}$ ,    $b = 4800 \approx 5000\text{M}^{-1}\text{s}$

Slope changes  $\Rightarrow$  **Competitive inhibition**

$$\frac{1}{v} = \frac{K_S}{k_2[E]_0} \left( 1 + \frac{k_3}{k_{-3}}[I] \right) \frac{1}{[S]} + \frac{1}{k_2[E]_0}$$

where

Ratio of the slopes:

$$a = \frac{K_S}{k_2[E]_0} \left( 1 + \frac{k_3}{k_{-3}}[I] \right)$$

$$b = \frac{1}{k_2[E]_0}$$

$$\frac{11.5}{4.0} = \frac{a}{a_0} = 1 + \frac{k_3}{k_{-3}}[I] \quad \Rightarrow \quad \frac{k_3}{k_{-3}} = \frac{\frac{11.5}{4} - 1}{[I]} = 9400\text{M}^{-1}$$

$y$  intercept:

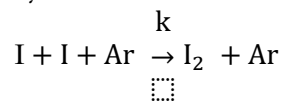
$$b_0 = \frac{1}{k_2[E]_0} \Rightarrow k_2 = \frac{1}{b_0[E]_0} = \frac{1}{5000 \times 10^{-5}} \text{s}^{-1} = 20 \text{s}^{-1}$$

Slope:

$$a_0 = \frac{K_S}{k_2[E]_0} \Rightarrow K_S = K_M = \frac{a_0}{b_0} = \frac{4.0}{5000} \text{M} = 8 \times 10^{-4} \text{M}$$

## Problem 2

The recombination of iodine atoms in the presence of argon is a third order reaction, that has been extensively studied using techniques such as flash photolysis.



a) In an experiment, the concentration of argon is  $1 \times 10^{-2}$  mol/l, and the initial concentration of iodine atoms is  $6 \times 10^{-5}$  mol/l. At a temperature of 298 K, the half-life of the iodine atoms is 238  $\mu\text{s}$ . Calculate the rate constant.

The rate equation is

$$v = -\frac{1}{2} \frac{d[\text{I}]}{dt} = k [\text{I}]^2 [\text{Ar}]$$

which we can integrate ( $[\text{Ar}] = \text{const}$ )

$$\int_{[\text{I}]_0}^{[\text{I}]} \frac{d[\text{I}]}{[\text{I}]^2} = -2k[\text{Ar}] \int_0^t dt$$

to obtain

$$t = \frac{\frac{1}{[\text{I}]} - \frac{1}{[\text{I}]_0}}{2k[\text{Ar}]}$$

With

$$[\text{I}](t = t_1) = \frac{1}{2} [\text{I}]_0$$

we find

$$t_{\frac{1}{2}} = \frac{1}{2k[\text{Ar}][\text{I}]_0}$$

With  $[\text{Ar}] = 1 \cdot 10^{-2} \text{ mol/l}$ ,  $[\text{I}]_0 = 6 \cdot 10^{-5} \text{ mol/l}$  and  $t_{\frac{1}{2}, 298 \text{ K}} = 238 \text{ }\mu\text{s}$ , we obtain a rate constant

$$k_{298 \text{ K}} = \frac{1}{2t_{\frac{1}{2}, 298 \text{ K}}[\text{Ar}][\text{I}]_0} = 3.50 \cdot 10^9 \frac{\text{l}^2}{\text{mol}^2 \text{s}}$$

b) At a temperature of 350 K, but otherwise identical initial conditions as in a), the half-life of the iodine atoms is 342  $\mu\text{s}$ . Calculate the activation energy of the reaction.

For  $t_{\frac{1}{2}, 350 \text{ K}} = 342 \text{ }\mu\text{s}$ , the rate constant is

$$k_{350 \text{ K}} = \frac{1}{2t_{\frac{1}{2}, 350 \text{ K}}[\text{Ar}][\text{I}]_0} = 2.44 \frac{\text{l}^2}{\text{mol}^2 \text{s}}$$

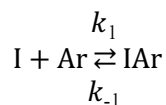
With the Arrhenius equation

$$k = A e^{-\frac{E_A}{RT}}$$

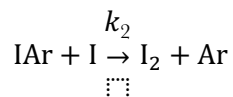
it follows that

$$E_A = \frac{\ln\left(\frac{k_{350 \text{ K}}}{k_{298 \text{ K}}}\right) R}{\frac{1}{298 \text{ K}} - \frac{1}{350 \text{ K}}} = -6.05 \frac{\text{kJ}}{\text{mol}}$$

c) The following reaction mechanism has been suggested for the recombination of iodine atoms. First, the van-der-Waals complex IAr is formed in a weakly exothermic reaction that is reversible.



In a second step, collision with another iodine atom leads to the formation of  $\text{I}_2$ .



The activation energy for this second step is zero. Since this second step is rate limiting, a pre-equilibrium exists for the formation of the complex IAr in the first step.

Write down the rate equation for the formation of  $I_2$  under the assumption of a pre-equilibrium for the complex IAr and explain why the recombination of iodine atoms has a negative activation energy. Hint: Consider the temperature dependence of the effective rate constant.

The rate equation for the pre-equilibrium is

$$v = \frac{d[I_2]}{dt} = k_2 K [I]^2 [Ar]$$

with

$$K = \frac{k_1}{k_{-1}}$$

We thus obtain an effective rate constant for the reaction

$$k_{\text{effective}} = k_2 K$$

The temperature dependence of  $k_2 = A e^{-\frac{E_a}{RT}} = A$  is small. In contrast, the equilibrium is increasingly shifted to the reactant side with increasing temperature since  $K = e^{-\frac{\Delta G^0}{RT}}$  and  $\Delta G^0$  is negative. This leads to a negative activation energy.

$$E_a = RT^2 \frac{d \ln k}{dT} \approx \Delta G^0$$